7.2 Compound Angle Formulas

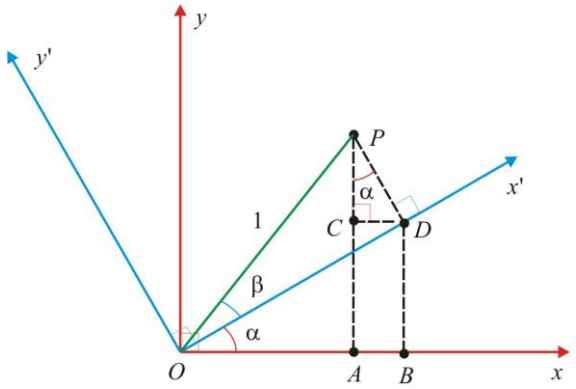
A Addition Formulas

The following relations are trigonometric identities called addition formulas:

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$
$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

Ex 1. Use the addition formulas for $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$ to prove the addition formula for $\tan(\alpha+\beta)$.

Proof:



B Subtraction Formulas

The following relations are trigonometric identities called subtraction formulas:

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$
$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$
$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

Ex 2. Replace in the addition formulas β by $-\beta$, then use the symmetry of the sine and cosine functions and the addition formula to prove the subtraction formulas.

Ex 3. Use $\sin(\pi/2)=1$ and $\cos(\pi/2)=0$ to prove the following formulas:

$$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$$
$$\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$$
$$\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$$

Ex 4. Use $\sin(\pi/2)=1$ and $\cos(\pi/2)=0$ to prove the following formulas: $\sin\left(\frac{\pi}{2}+\theta\right)=\cos\theta$ $\cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta$ $\tan\left(\frac{\pi}{2}+\theta\right)=-\cot\theta$	
Ex 5. Use $\sin(\pi)=0$ and $\cos(\pi)=-1$ to prove the following formulas: $\sin(\pi-\theta)=\sin\theta$ $\cos(\pi-\theta)=-\cos\theta$ $\tan(\pi-\theta)=-\tan\theta$	
Ex 6. Use $\sin(\pi)=0$ and $\cos(\pi)=-1$ to prove the following formulas: $\sin(\pi+\theta)=-\sin\theta$ $\cos(\pi+\theta)=-\cos\theta$ $\tan(\pi+\theta)=\tan\theta$	
Ex 7. Use $\sin(3\pi/2)=-1$ and $\cos(3\pi/2)=0$ to prove the following formulas: $\sin\left(\frac{3\pi}{2}-\theta\right)=-\cos\theta$ $\cos\left(\frac{3\pi}{2}-\theta\right)=-\sin\theta$ $\tan\left(\frac{3\pi}{2}-\theta\right)=\cot\theta$	
Ex 8. Use $\sin(3\pi/2)=-1$ and $\cos(3\pi/2)=0$ to prove the following formulas: $\sin\left(\frac{3\pi}{2}+\theta\right)=-\cos\theta$ $\cos\left(\frac{3\pi}{2}+\theta\right)=\sin\theta$ $\tan\left(\frac{3\pi}{2}+\theta\right)=-\cot\theta$	
Ex 9. Use $\sin(\pi)=0$ and $\cos(\pi)=-1$ to prove the following formulas: $\sin(2\pi-\theta)=-\sin\theta$ $\cos(2\pi-\theta)=\cos\theta$ $\tan(2\pi-\theta)=-\tan\theta$	

Ex 10. Find the exact value of $\sin\frac{5\pi}{12}$.

Ex 11. Find the exact value of $\cos\frac{\pi}{12}$.

Ex 12. Find the exact value of $\tan\frac{7\pi}{12}$.

Ex 13. Graph $f(x)=\frac{\sin x+\cos x}{\sin x-\cos x}$.

Ex 14. Prove that:

$$\cot(\alpha+\beta)=\frac{\cot\alpha\cot\beta-1}{\cot\alpha+\cot\beta}$$

Ex 15. Prove that:

$$\csc(\alpha+\beta)=\frac{\csc\alpha\csc\beta}{\cot\alpha+\cot\beta}$$

Reading: Nelson Textbook, Pages 394-399
Homework: Nelson Textbook, Page 400: #2a, 5abc, 8ade, 9bcf, 12b, 16, 17

